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Entangling Two-Atom Through Cooperative Interaction Under Stimulated Emission

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We discuss the generation of two two-level atoms entanglement inside a resonant microcavity under stimulated emission (STE) interaction. The amount of entanglement is studied based on different atomic initial states. For each kind of initial state, we obtain the entanglement period and the entanglement critical point, which are found to deeply depend on driving field density. If both atoms are initially in excited state, the entanglement can be induced due to STE. While when one of them initially lies in its ground state, we find there is a competition between driving field induced entanglement and STE induced entanglement.

KEY WORDS: cooperative interaction; stimulated emission; concurrence.

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1. INTRODUCTION

One of the most interesting features of quantum mechanics is the correlation between pairwise-entangled quantum states of two spatially separated particles, which is called EPR pair. Besides the applications of EPR pairs on investigating the conceptual foundations of quantum mechanics, such as testing the violation of Bell inequality, a great deal of interesting has been intensively focused on designing and realizing possible quantum entangling proposals that can be essential ingredients in quantum communication (Bennett and Brassard, 1984; Vedral, 2002; Pan *et al.*, 2003; Zhao *et al.*, 2004) and quantum computation (Nielsen and Chuang, 2000). These EPR pairs can be formatted in different physical systems such as trapped ions (Sackett *et al.*, 2000), spins in nuclear magnetic resonance (Li *et al.*, 2003), superconductor Josephsen junctions (Berkley *et al.*, 2002), Cooper pairs in solid states quantum-dots (Stievater *et al.*, 2001; Kamada *et al.*, 2001), and cavity quantum-electrodynamics systems (CQED) (Rauschenbeutel *et al.*, 2000). Among these systems, CQED system has been deeply studied for entangling two atoms or

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two modes field in constructing quantum logic gates (Zheng and Guo, 2000; Pachos and Walther, 2002; Biswas and Agarwal, 2004) or quantum memory (Schori *et al.*, 2002; Duan *et al.*, 2002). Practically, two atoms can be entangled through continuously driving by a coherent pump field (Plenio *et al.*, 1999; Zheng and Guo, 2000), the assistance of a thermal cavity field (Kim *et al.*, 2002; Zhou *et al.*, 2002; Yi *et al.*, 2003; Zhou *et al.*, 2004), or even the inducement from the atomic spontaneous emission (Yi *et al.*, 2003; Jakobczyk and Jamroz, 2003; Guo *et al.*, 2004; Ficek and Tanaś, 2003). Generally, these schemes are effective since there are some kinds of interaction between atom and atom or atom and cavity field. While, there is another interaction that is usually not included: the stimulated emission (STE, which refers to Einstein B coefficient (Mompart and Corbalán, 2001)) of atoms in a atomic ensemble. The STE of atom emerges when atom in higher energy level is driven by a polarized photon (Konôpka, 1999). Especially when atomic population inversion is realized, in a laser system, STE plays a key role in photon absorb-emission process. In Tan and Gu (1985), taking into account the STE, the authors study the resonance fluorescence spectrum and present five peaks are formed due to STE. In solving the resonance fluorescent spectrum, the authors treat the emitted photon as a new driving field acting on both atoms since the emitted photon has the identical character with that of driving photon. In this paper, we consider a system with this interaction and try to analyze the entanglement character of two-atom. The system is described in the first section. In second section, the measurement of entanglement is presented. And in last section, some results on two-atom entanglement nature are obtained.

2. COOPERATIVE INTERACTION BETWEEN ATOMS

We consider a system constituted by two two-level atoms located in a microcavity which is located in vacuum and a single mode quantized cavity field. Figure 1 shows the schematic diagram of this system. Either atom can transit from excited state $|e\rangle$ to ground state $|g\rangle$ under the driving of a resonant laser field and emit a polarized photon. Also, either atom that is in state $|g\rangle$ can absorb such polarized photon and jump to state $|e\rangle$. That is, besides driving laser field E, the emitted polarized photon from one atom can also act as a new driving laser field E' with respect to the other atom, and vice versa. As a result, the whole cavity field is the sum of two fields E and E'. In Tan and Gu (1985), the authors discuss a similar model where they assume the atoms are coupled to a classical external field. In fact, their model is effective in case of a quantized driving field. The Hamiltonian of the system in the interaction picture thus reads

$$H = g_{drv} \sum_i (a\sigma_i^+ + a^+\sigma_i^-) + g_{ste}/2 \sum_{i,j} [\sigma_i^z (a\sigma_j^+ + a^+\sigma_j^-) + (a\sigma_j^+ + a^+\sigma_j^-)\sigma_i^z]. \tag{1}$$

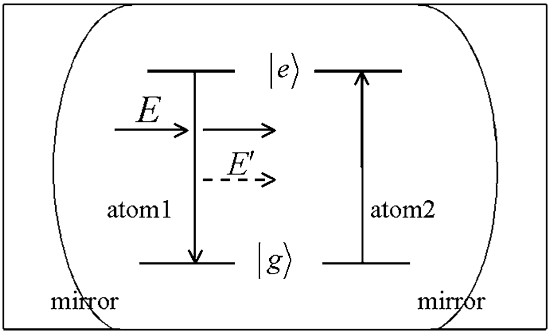

Fig. 1. Schematic diagram for two two-level atoms system cooperative interaction through STE. Under the drive of a laser field E, one excited atom can fall to its ground state. The emitted photon acts as a new field E' acting on another atom, and vice versa.

The upper Hamiltonian is equivalent with

$$H = \left(g_{drv} + g_{ste} \sum_i \sigma_i^z\right) \sum_j (a\sigma_j^+ + a^+\sigma_j^-). \tag{2}$$

We are familiar with the form of this Hamiltonian since it should be a system with two atoms directly interact with a cavity field. In (2), a, a^+ are eliminate and create operators of driving field, $\sigma_i^+ = |e\rangle_i\langle g|$, $\sigma_i^- = |g\rangle_i\langle e|$ are transition operators of atom i and $\sigma_i^z = \frac{1}{2}(|e\rangle_i\langle e| - |g\rangle_i\langle g|)$ is the inversion operator of atom i, g_{drv} represents the coupling strength between separate atom and field E. g_{ste} denotes the interaction strength between atom and j and E'. Generally, $g_{ste} = \alpha_s g_{drv}$ and α_s is determined by STE coefficient, atomic density (which can be altered by changing the size of the cavity, even this is difficult to achieve) and g_{drv}. When two atoms are spatially much close to each other, α_s will fall in the range between 0 and 1 (Tan and Gu, 1985). The second sum for subscripts i and j proceeds for $i, j = 1, 2$ and $i \neq j$. From (2), the unitary operator that governs the evolution of the density matrix of global system with initial state $\rho(0)$ can be obtained as $\hat{U}(t) = e^{-iHt/\hbar}$. Formally, the evolved system density is $\rho(t) = \hat{U}(t)\rho(0)\hat{U}^{\dagger}(t)$. In the basis of $|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle$, by expanding $\hat{U}(t)$ into a combination of Taylor series, the evolution operator matrix is given by

$$U = \begin{pmatrix} 2g^2a\Theta a^+ + 1 & -iga\Phi & -iga\Phi & 2g^2\gamma a\Theta a \\ -ig\frac{\sin\Omega t}{\Omega}a^+ & \frac{1}{2}(\cos\Omega t + 1) & \frac{1}{2}(\cos\Omega t - 1) & -ig\gamma\Phi a \\ -ig\frac{\sin\Omega t}{\Omega}a^+ & \frac{1}{2}(\cos\Omega t - 1) & \frac{1}{2}(\cos\Omega t + 1) & -ig\gamma\Phi a \\ 2g^2\gamma a^+\Theta a^+ & -ig\gamma a^+\Phi & -ig\gamma a^+\Phi & 2g^2\gamma^2 a^+\Theta a + 1 \end{pmatrix}, \tag{3}$$

where, we have set $\Theta = \frac{\cos\Omega t - 1}{\Omega}$, $\Phi = \frac{\sin\Omega t}{\Omega}$, $\Omega = \{2g^2[(\gamma^2+1)a^+a+\gamma^2]\}^{\frac{1}{2}}$, $g = (1+\alpha_s)g_{drv}$ and $\gamma = \frac{1-\alpha_s}{1+\alpha_s}$. What attracts us is the evolution process of the two-atom sub-system density matrix which is obtained by tracing over the field variables of system density matrix $\rho(t)$. We expect the cooperative interaction can induce atom–atom entanglement during the evolution.

3. MEASUREMENT OF ENTANGLEMENT DEGREE

Using (3), we can easily find that whatever be the initial state of two-atom, the time evolution operator $\hat{U}(t)$ would reduce most of the off-diagonal elements of the density matrix (Kim *et al.*, 2002). The resulting two-atom density matrix can be written as

$$\rho_a(t) = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & E & 0 \\ 0 & E & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}, \tag{4}$$

Using the entanglement degree defined by Wootters concurrence simplified from the entanglement of formation (Hill and Wootters, 1997; Wootters, 1998)

$$C = \max\left\{0, 2\max\{\lambda_i\} - \sum_i \lambda_i\right\}, \tag{5}$$

where λ_i are the four non-negative square roots of the eigenvalues of the non-Hermitian matrix $\rho_a(t)\tilde{\rho}_a(t)$ with $\tilde{\rho}_a(t) = (\sigma_y \otimes \sigma_y)\rho_a^*(t)(\sigma_y \otimes \sigma_y)$. We obtain

$$C(\rho_a) = 2\left(\min\{E, \sqrt{B\cdot C}\} - \sqrt{A\cdot D}\right). \tag{6}$$

Alternatively, the two-atom entanglement can be measured by the criteria defined as the partial transposition proposed by Peres and Horodecki which is written as $\varepsilon = -2\sum_i \mu_i$ with μ_i corresponding to the negative eigenvalues of the partial transposition $\rho_a^T(t)$ of density matrix. It has been discussed in (Zhou *et al.*, 2002, 2004) that only when $E^2 > A\cdot D$ may the entanglement of two-atom be created. This criteria for entanglement is equivalent with Concurrence when $E^2 \leq B\cdot C$ (in fact, the equality is obvious in the following results). Both criteria are suitable for measuring the entanglement of arbitrary two qubits system whether the system being pure state or mixed one. Here, we use Concurrence as the entanglement measurement.

4. ATOM–ATOM ENTANGLEMENT DISCUSSION

We assume the initial cavity driving field is in single mode Fork state $|n\rangle_f$ with photon number n presenting the cavity field density. So that the reduced two-atom sub-system density matrix is

$$\begin{aligned}\rho_a(t) &= Tr_f\rho(t) = Tr_f\big[\hat{U}(t)\rho(0)\hat{U}^\dagger(t)\big] \\ &= \sum_m {}_f\big\langle m\big|\hat{U}(t)\big|n\big\rangle_f \rho_a(0)_f\big\langle n\big|\hat{U}^\dagger(t)\big|m\big\rangle_f \end{aligned} \tag{7}$$

where $\rho_a(0)$ is the initial state of two-atom subsystem. The matrix element ${}_f\langle m|\hat{U}(t)|n\rangle_f$ presents the influence of cavity mode on the atomic transition. In the following analysis, we will present the entanglement nature of two atoms under STE when they are initially in different states. For convenience, we set $g \equiv 1$ in following calculation. Firstly, we consider two atoms are both initially in their excited state, that is $\rho_a(0) = |e\rangle_{11}\langle e| \otimes |e\rangle_{22}\langle e|$. We get, in (5), $A = |U_{11}|^2 = [1 + 2(n+1)\frac{\cos g\xi t - 1}{\xi^2}]^2$, $B = C = E = |U_{21}|^2 = (n+1)\frac{\sin^2 g\xi t}{\xi^2}$, $D = |U_{41}|^2 = 4\gamma^2(n+1)(n+2)\frac{(\cos g\xi t - 1)^2}{\xi^4}$, where $\xi = \sqrt{2[(\gamma^2+1)(n+1)+\gamma^2]}$ where we define $U_{ij} = \sum_m \langle m|\hat{U}_{ij}(t)|n\rangle$. It has been be stressed in (Kim *et al.*, 2002) that no two-atom entanglement can be generated when two atoms are initially in $|e\rangle_1|e\rangle_2$ no matter what state the cavity mode is in. While, when STE is included, the result is apparently different. We can find that the necessary and sufficient condition of generating positive Concurrence in (8) is $0 \le \gamma < \sqrt{\frac{n+1}{n+2}}$. That is, there exists a critical point $\gamma_0 = \sqrt{\frac{n+1}{n+2}}$ ($g_{ste,crit} = (\sqrt{n+2} - \sqrt{n+1})^2 g_{drv}$) which turns out to be the minimum value of STE coupling strength for generating two-atom entanglement that are initially in excited states. One of the ways to decrease the critical point is increasing the density of field. Extremely, for large n, this point tends to zero, which means, in a high field density cavity, even a slight STE can induce two-atom entanglement. For a vacuum field, this point is about $0.17 g_{drv}$. To show these properties, we plot two-atom entanglement as a function of Time-t and γ in Figs. 2 and 3 with different driving field density n. Both figures indicate that the Concurrence is almost a monotone decreasing function of γ. Along t axis, the concurrence presents a sine-square-like behavior with periodical maximum and minimum-zero. While, this period can be changed by alternating γ. Only when $g_{ste} = g_{drv}$ does the Concurrence act as an exactly sine-square function $\sin^2\sqrt{2(n+1)}t$, where the period presents as $\frac{\pi}{\sqrt{2(n+1)}}$ which is, for example, exactly $\frac{\pi}{2}$ for $n = 0$ and $\frac{\sqrt{2}\pi}{4}$ for $n = 1$. Elsewhere, the period of entanglement along t axis is $\frac{2\pi}{\xi}$ for a given γ. It is fascinating that the driving field density n not only determines the critical value of STE coupling strength, but also takes great influence on the entanglement-disentanglement period. Generally, the larger the driving field density the smaller the entanglement period.

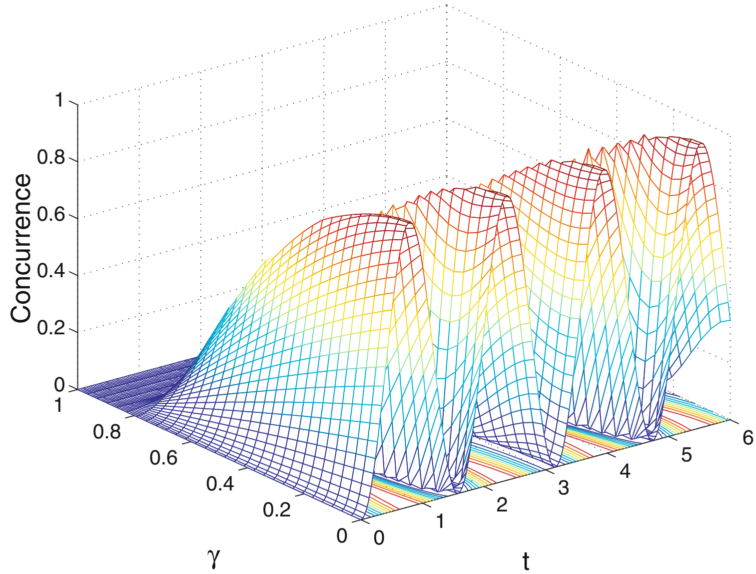

Fig. 2. Two-atom (initially in $|e, e\rangle$) entanglement as a function of t and γ for $n = 1$.

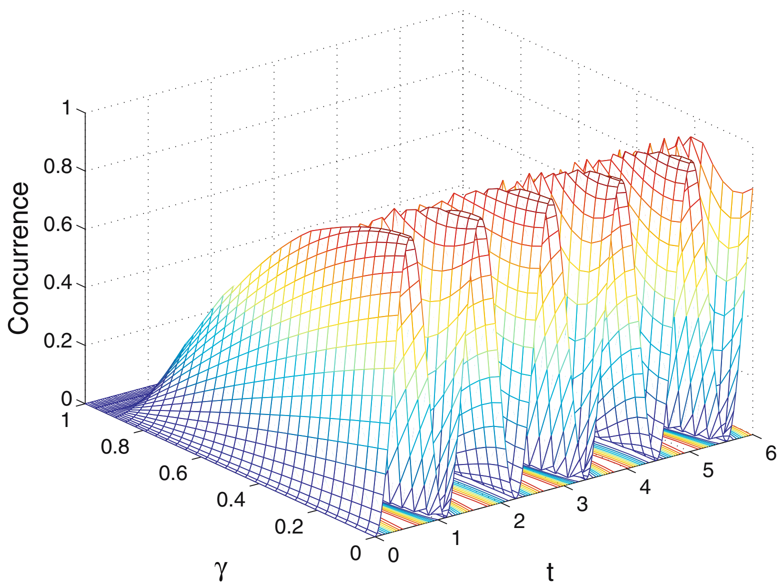

Fig. 3. Two-atom (initially in $|e, e\rangle$) entanglement as a function of t and γ for $n = 3$.

Another unapparent character of two figures is the peak of the Concurrence for a same γ can be increased by increasing field density except for the range of γ very close to zero. This can be easily understood since $g_{ste,crit}$ can be increased by increasing n. In constructing practical quantum logical gates, we may need strong and long sustained entanglement, thus, we should make a suitable choice of controllable physical parameters such as the STE coupling strength and the density of monochromatic driving field. Secondly, we assume one of the atoms is initially excited and the other has falled to its ground state, thus the two-atom sub-system initial state is $\rho_s(0) = |e\rangle_{11}\langle e| \otimes |g\rangle_{22}\langle g|$. The resulting two-atom density matrix elements can be obtained as $A = |U_{12}|^2 = n\frac{\sin^2 g\xi t}{\xi^2}$, $B = |U_{22}|^2 = (\cos g\xi t + 1)^2/4$, $C = |U_{23}|^2 = (\cos g\xi t - 1)^2/4$, $D = |U_{42}|^2 = \gamma^2(n+1)\frac{\sin^2 g\xi t}{\xi^2}$, $E = |U_{22}U_{23}| = \sin^2 g\xi t/4$, where $\xi = \sqrt{2[(\gamma^2+1)n+\gamma^2]}$. We can also find the necessary and sufficient condition of generating positive entanglement is $\gamma \neq \sqrt{\frac{n}{n+1}}$. The critical point is $\gamma_0 = \sqrt{\frac{n}{n+1}}$ ($g_{ste,crit} = \left(\sqrt{n+1} - \sqrt{n}\right)^2 g_{drv}$) which can be shifted towards zero by increasing n. The entanglement situation is shown in Figs. 4 and 5. The whole area can be separated into two regions: the region where $0 \leq \gamma < \gamma_0$ and the region where $\gamma_0 < \gamma \leq 1$. In the first region, the entanglement increases rapidly with respect to γ. Especially, in the area that $\gamma \to 0$, where two coupling strengthes g_{drv} and g_{ste} are

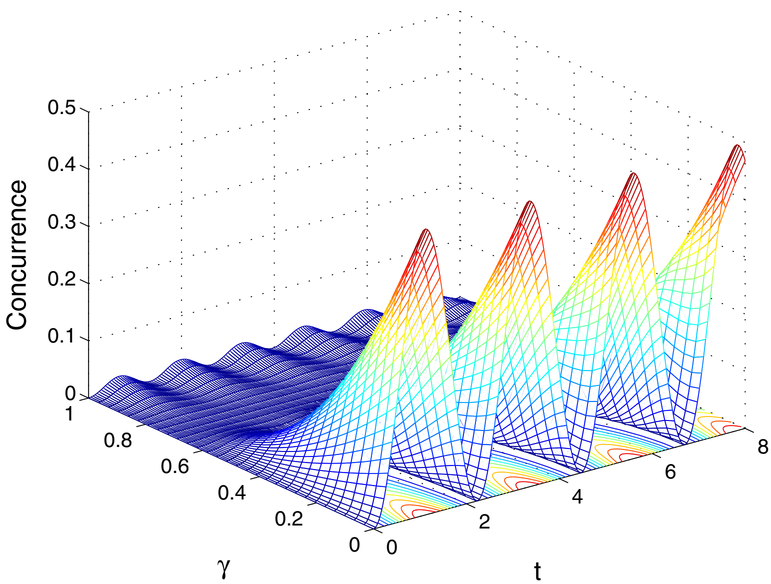

Fig. 4. Two-atom (initially in $|e, g\rangle$) entanglement as a function of t and γ for $n = 3$.

comparable, the entanglement monotonously reaches its peak. While, it can be easily proved that this peak can never exceed 0.5 which is the most probable maximum value of entanglement. For a given γ, the time evolution of entanglement presents periodical loss and revival with a period $\frac{\pi}{\xi}$. Obviously, to obtain long time sustained entanglement, STE should be outstanding and driving field density should not be large. When $n \to 0$ and $\gamma \to 0$, the period tends to infinite! Under this circumstance, large entanglement can never be obtained in finite time. In the second region, where STE is very weak, the coupling between atom and driving field is dominating. The result is similar with that obtained (Kim *et al.*, 2002). But the resulting entanglement is much weak (see Fig. 4) and only emerges when driving field density is small (see Fig. 5). In both regions, the loss and revival of the entanglement also show to be periodical. When n tends to zero, the period, which is approximately $\frac{\pi}{\sqrt{2}\gamma}$, is only dependent on difference between two coupling strengthes γ. It should be stressed that even when driving field density is large, and the first-term interaction in (2) can hardly induce entanglement, STE can generate entanglement. To sum up, we see there is a competition, which depends on n and γ, between first-term interaction and second-term interaction in (2). The competition behaves with: which is the domination of entanglement, how about the shift of the critical point and what is the contrast of entanglement periods in two regions. While, whatever be the competition, apparent STE can enhance two-atom entanglement.

5. CONCLUSION

We have discussed the generation of two-atom entanglement inside a resonant microcavity under an auxiliary interaction-STE. The entanglement when two atoms are initially in $|e\rangle_1 \, |e\rangle_2$ and $|e\rangle_1 \, |g\rangle_2$ is studied. Some meaningful results are obtained with the assistance of different STE. The induced entanglement is simply determined by an analytic sine-like function of time and difference between two coefficients. In both cases, we get the critical points of generating entanglement as well as the entanglement period. These two quantities can both be controlled by adjusting the density of driving field or STE coefficient in experiment scenario. We have created two-atom entanglement in first case where it was shown impossible generating entanglement without STE. In second case, we find a competition between the interaction with and without STE, while, the amplitude of entanglement with STE is much larger than that without STE, so is the period. While, in dealing with this system, we do not take into account the atomic spontaneous emission which leads to a line width of the emitted photon. Also, we do not include the dissipation of the cavity that has been considered in many papers (Plenio, 1999; Nicolosi, 2004). Even after including these effects, the STE could still plays an important role in a collectively excited atomic ensemble. The entanglement in-

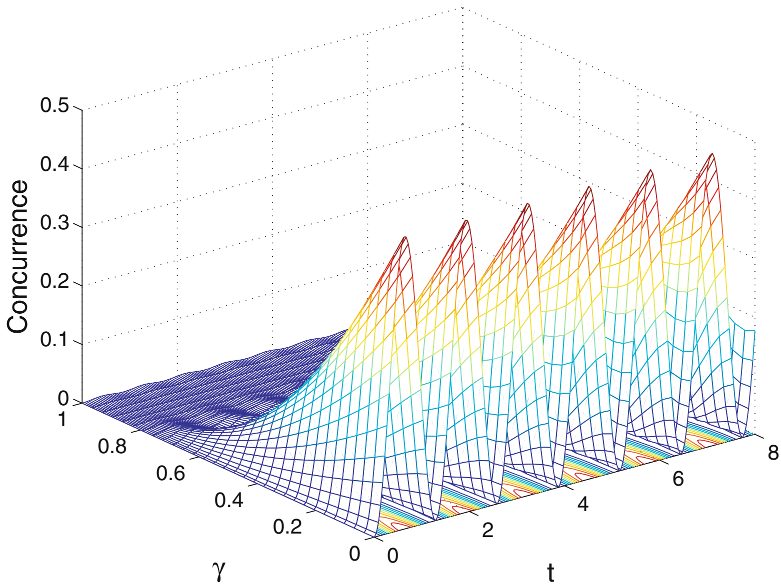

Fig. 5. Two-atom (initially in $|e, g\rangle$) entanglement as a function of t and γ for $n = 1$.

duced by stimulated emission should also be considered when atoms are used to construct quantum logic gate or storage photon information. Though we only studied two atoms in a large number of an atomic ensemble, the results can be extended to a multi-atom system.

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